Economic Dispatch with Convex and Non-Convex Fuel Cost Functions Including Line Losses Using PS and GA Approaches

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Abstract - **This article presents an application of Generalized Patten Search (PS) and Genetic Algorithm (GA) to solve Economic Load Dispatch (ELD) problems with convex and non-convex fuel cost objective functions. Main objective is to determine the most economic generating dispatch required to satisfy the predicted load demands including line losses over a certain period of time while relaxing various equality and inequality constraints. The unit operational minimum/maximum constraints, effects of valve-point ripples and line losses are considered for the practical applications. Several cases were tested and verified, which indicate an improvement in total fuel cost savings. The robustness of the proposed methods have been assessed and investigated through comparisons with results reported in recent researches. The results are very encouraging and suggesting that both PS and GA are very useful in solving power system ELD problems efficiently with global minimum fuel cost.**

Keywords - **Pattern Search (PS), Genetic Algorithm (GA), Economic Load Dispatch, Valve-point effects, Optimal Solution.**

I. INTRODUCTION

ELD is the short-term determination of the optimal output of a number of electricity generation facilities, to meet the system load, at the lowest possible cost, while serving power to the public in a robust and reliable manner [1-3].

Many recent works have been around Artificial Intelligence (AI) methods, on par with the development of AI optimization theories, such as Artificial Neural Networks (ANN), Simulated Annealing (SA), Genetic Algorithms (GA), Differential Evolution (DE), Particle Swarm Optimization (PSO), Evolutionary Programming (EP), and hybrid methods [3-6]. ELD algorithms for thermal unit system involving combined cycle units presented in [7]. Online solving of economic dispatch problem using neural network approach and comparing it with classical methods were presented in [8]. The Evolutionary Algorithms (EAs) are different from the conventional optimization methods, and they do not need to differentiate cost function and constraints. Theoretically, like simulated annealing, EAs converge to the global optimum solution. EAs, including EP, Evolutionary Strategy (ES), and GA are artificial intelligence methods for optimization based on the mechanics of natural selection, such as mutation, recombination, reproduction, crossover, selection, etc [9-11]. Many researchers exert to improve many optimization techniques for solving ELD problem such as PSO [12-16], GA [17-18], Hopfield solution [20] and SA [19]. Moreover, little work was exerted with PS [26]. Direct Search (DS) and GA methods, as opposed to more standard optimization methods, are often called derivative-free as they do not require any information about the gradient or higher derivatives of the objective function to search for an optimal solution. Therefore, DS and GA methods may very well be used to solve non-continuous, non-differentiable and multimodal optimization problems [4-6, 25-27]. PS has the advantage of being very simple in concept, and easy to implement and computationally efficient algorithm. On the other hand, the PS is more sensitive to the initial guess and appears to rely on how close the given initial point is to the global solution and normally generates single point at each iteration. The later makes the PS method possibly more susceptible to getting trapped in local minima. However, GA generates a population of points at each iteration and hence, the best point in the population approaches an optimal solution. Nevertheless, both GA and PS can be considered as integrating approaches rather than competitive methods.

In this work, the Generalized PS method and GA have been employed and utilized to solve ELD problems with a considering valve-point effects and line losses. A valve-point effect is the rippling effect added to the generating unit curve when each steam admission valve in a turbine starts to open. The data of 6 generating units and 40 generating units have taken to which are numerical tested and compared.

II. ELD PROBLEM FORMULATION

The primary concern of an ELD problem is the minimization of its objective function. The total cost generated that meets the demand and satisfies all other constraints associated is selected as the objective function [1, 3]. In general, the ELD problem can be formulated mathematically as a constrained optimization problem with an objective function of the form, as in Eq. (1):

Objective Function:

$$\textbf{\textit{Minimize:}}\ FC_T = \sum_{i=1}^{N} FC_i(P_i) \tag{1}$$

Where FC_T is the total generation cost; N is the total number of generating units; FC_i is the power generation cost function of the i[th] unit.

A. Classical Smooth Fuel Cost Functions

Generally, the fuel cost of a thermal generation unit is considered as a second order polynomial function (Neglecting valve-point effects) and this is called classical and smoothed cost function (refer to Eq. (2)).

$$FC_i(P_i) = a_i P_i^2 + b_i P_i + c_i \quad (2)$$

Where P_i is the power of the i^{th} generating unit; a_i, b_i, c_i are the fuel cost coefficients of the i^{th} generating unit.

B. Non-Smooth Fuel Cost Functions including Valve-point Loading Effects

The generating units with multi-valve steam turbines exhibit a greater variation in the fuel cost functions. Since the valve point results in the ripples [22-23], a cost function contains higher order nonlinearity. Therefore, the cost function should be modified to consider the valve-point effects as shown in fig. 1. Typically, the valve point results in, as each steam valve starts to open, the ripples like in to take account for the valve-point effects, sinusoidal functions are added to the quadratic cost functions as follows in Eq. (3) [24]:

$$FC_i(P_i) = a_i P_i^2 + b_i P_i + c_i + \left| e_i \times \sin\left(\left(f_i \times \left(P_{i,min} - P_i\right)\right)\right)\right| \quad (3)$$

Where a_i, b_i, c_i, e_i and f_i are the cost coefficients of generator (i) reflecting valve-point loading effects.

This model either with smooth or non-smoothed fuel cost functions is subjected to the following constraints:

Equality constraint:

The total generated power should be equal to total load demand plus the total losses, as in Eq. (4).

$$\sum_{i=1}^{N} P_i = P_{Demand} + P_{Loss} \quad (4)$$

Where P_{Demand} is the total system load demand and P_{Loss} is the total line losses. Total line losses P_{Loss} can be calculated using Eq. (5).

$$P_{Loss} = \sum_{i=1}^{N}\sum_{j=1}^{N} P_i^T B_{ij} P_j + \sum_{i=1}^{N} P_i B_{oi} + B_{oo} \quad (5)$$

B_{ij} , B_{oi} and B_{oo} are transmission line loss coefficients (P_i^T is vector transpose of all generation plants net MW, Bij is square matrix of same dimension as P_i and B_{oi} is vector of same length as P_i and B_{00} is constant).

Inequality constraints for each generator are shown in Eq. (6);

$$P_{i,min} \leq P_i \leq P_{i,max} \text{ for i=1, 2, …., NG} \quad (6)$$

Where P_i is the output power of generator i; $P_{i,,min}$ and $P_{i,,max}$ are the min/max power outputs of generator i.

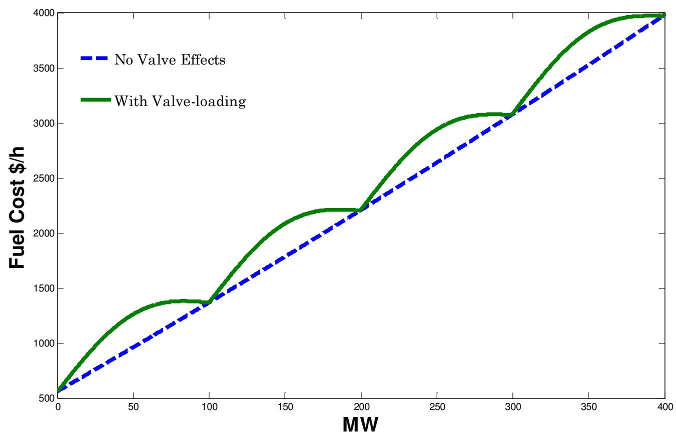

Fig. 1 Fuel Cost function with and without valve-effects

III. PS OPTIMIZATION

Unlike more traditional optimization methods that use information about the gradient or higher derivatives to search for an optimal point, a direct search algorithm searches a set of points around the current point, looking for one where the value of the objective function is lower than the value at the current point. Generally, PS has the advantage of being very simple in concept, and easy to implement and computationally efficient algorithm. On the other hand, the PS is more sensitive to the initial guess and appears to rely on how close the given initial point is to the global solution. This makes the PS method possibly more susceptible to getting trapped in local minima. However, the much improved speed of computation allows for additional searches to be made to increase the confidence in the solution. PS includes two direct search algorithms called the generalized pattern search (GPS) algorithm and the mesh adaptive search (MADS) algorithm [24]. Both are pattern search algorithms that compute a sequence of points that approach an optimal point. At each step, the algorithm searches a set of points, called a mesh, around the current point - the point computed at the previous step of the algorithm. A major aspect of any GPS algorithm is the rule for generating the meshes on which the searches are conducted. The PS optimization routine is an evolutionary technique that is suitable to solve a variety of optimization problems that lie outside the scope of the standard optimization methods. Direct search methods can solve constrained and unconstrained optimization problems [27]. Different search functions are available and moreover, can be customised if needed.

A. Generation of the Meshes

The PS algorithm proceeds by computing a sequence of points that may or may not approaches to the optimal point. The algorithm starts by establishing a set of points called mesh, around the given point. This current point could be the initial starting point supplied by the user or it could be computed from the previous step of the algorithm. The mesh is formed by adding the current point to a scalar multiple of a set of vectors called a pattern. If a point in the mesh is found to improve the objective function at the current point, the new point becomes the current point at the next iteration. At each step, the algorithm polls the points in the current mesh by computing their objective function values.

B. Termination of Running PS Algorithm

The PS optimization algorithm will repeat the illustrated steps until it finds the optimal solution for the minimization of the objective function. The algorithm stops when any of the following conditions occurs:

- The mesh size is less than mesh tolerance.
- The number of iterations performed by the algorithm reaches a predefined value.
- The total number of objective function evaluations performed by the algorithm reaches a pre-set maximum number of function evaluations.
- The distance between the point found at one successful poll and the point found at the next successful poll is less than a set tolerance.
- The change in the objective function from one successful poll to the next successful poll is less than a function tolerance.

IV. OVERVIEW OF GA

GAs are well-known stochastic methods of global optimization based on the evolution theory of Darwin [4-6]. The GA is a method for solving both constrained and unconstrained optimization problems that is based on natural selection, the process that drives biological evolution. The GA uses three main types of rules at each step to create the next generation from the current population:

- Selection rules select the individuals, called parents that contribute to the population at the next generation.
- Crossover rules combine two parents to form children for the next generation.
- Mutation rules apply random changes to individual parents to form children.

Crossover rate - this rate generally should be high, about 80~95%. (However some results show that for some problems crossover rate about 60% is the best).

Mutation rate - on the other side, mutation rate should be very low.

Population size - it may be surprising that a very big population size usually does not improve performance of GA (in meaning of speed of finding solution). A good population size is about 20–30, however sometimes sizes 50–100 are reported as best. Some research also shows that best population size depends on encoding, on size of encoded string. It means, if you have a chromosome with 32 bits, the population should be say 32, but surely two times more than the best population size for chromosome with 16 bits.

The advantages of GA can be summarized as:

1. Optimizes with continuous or discrete variable.
2. Simultaneously searches from a wide sampling of the cost surface.
3. Deals with a large number of variables.
4. Provides a list of optimum variables, not just a single solution.
5. Optimizes variables with extremely complex cost surfaces.
6. May encode the variables so that the optimization is done with the encoded variables and
7. Works with numerically generated data, experimental data, or analytical functions.

A. GA Solution Procedure

The GA repeatedly modifies a population of individual solutions. Some of the commonly used terminologies in GA are fitness function- which we want to minimize and population- an array of individuals. A generalized procedure for GA is summarized below.

8. The first step is to define the objective function and variables.
9. In the second step the GA parameters are selected and the population is initialized *randomly*.
10. After initialization the elite randomly and parents are selected based on the fitness value.
11. Once the parents are selected either mutation or crossover is performed form offspring.
12. The offspring is then inserted into the new population.
13. The last step is to check if the optimum solution has been achieved.

B. Terminating the Run of GA Script

This generational process is repeated until a termination condition has been reached. Common terminating conditions are: set number of iterations, set time reached, a cost that is lower than an acceptable minimum, set number of cost function evaluations, a best solution has not changed after a set number of iterations, or operator termination.

V. PS, GA AND ELD

To use PS and GA algorithms to solve ELD, the following parameters were needed for data input.

1. Minimal and Maximal power outputs of each unit.
2. B-matrix of line losses.
3. Coefficients of unit fuel cost function including coefficients of valve-point loading.
4. Total load demand.

The objective function, equality and inequality constraints were written in Matlab m-file. The ranges for the variables were set in the DS and GA toolbox of MATTLAB 7.10 version [24]. *PS performance is dramatically enhanced using GA as search method.*

VI. SIMULATION RESULTS

To verify the feasibility and efficiency of applying PS and GA to solve ELD with taking the effect of valve ripples into consideration. Several cases were tested and investigated, among of these, two cases will be presented. The PS and GA based algorithms are applied to solve the six-units with line losses and neglecting effects of valve-loading as case I. In case II, the algorithms were applied to Forty-units system with considering valve-point loading effects and neglecting line losses. Simulations were carried out using GA and DS toolbox of MATLAB®7.10 release 2010a version and executed on a LABTOP with Processor Intel® Core i5 CPU 2.40 GHz with a 4.0 GB of RAM with 32-bit operating system.

Case I: 6-Units System with line losses.

The Fuel cost characteristics of the six units in Rs/h and the unit operating min/max (MW) ranges are given in [21]. Table I shows the optimal scheduling of generators for the power demands of 700 MW and 800 MW. As seen in table 1, both PS and GA have provided efficient results with relaxing all equality and inequality constraints with less computation elapsed time in the favour of GA. While best fuel costs are obtained using PS approach. Moreover, GA algorithm needs lot of tunned parameters to get proper results. On other hand, PS requires very few adjustments.

TABLE I OPTIMAL SCHEDULING OF A SIX-UNIT SYSTEM BY PS AND GA METHODS.

Unit	P_D = 700	P_D = 700	P_D = 800	P_D = 800
P_1	27.30096	28.30265	32.67373	32.59982
P_2	15.61244	10.00012	15.81606	14.48327
P_3	120.31087	118.95503	141.66228	141.54462
P_4	116.77564	118.67259	131.31169	136.04102
P_5	226.83767	230.76035	252.37105	257.65794
P_6	212.40501	212.74092	251.55072	243.00401
P_{Loss}	**19.2426**	**19.4317**	**25.3855**	**25.3307**
Best FC (Rs/h)	**36,918.9**	**36,912.1**	**41,899.8**	**41,896.6**
Time (Sec)	**1.7825**	**3.465776**	**1.7439**	**3.749712**
Method	**GA**	**PS**	**GA**	**PS**

FC=Fuel Cost

Fig. 2 and 3 show the convergence graphs for 700 and 800 MW power demands using GA.

Case II: 40-Units System with valve-loading effects (neglecting line losses).

The data of cost coefficients and coefficients reflecting valve-point effects are given in [21-23]. Losses are neglected for sake of verifications and the case of load demand of 10,500 MW is considered as in [21-22]. Table II shows self explanatory numerical results obtained with PS Optimization in comparisons with GA.

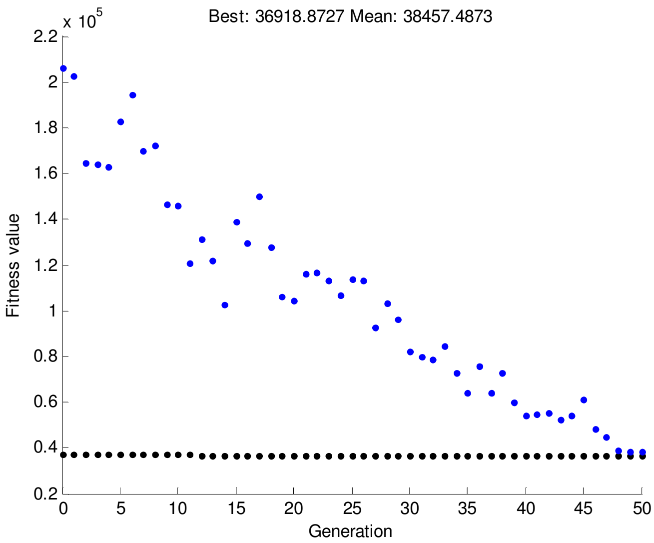

Fig. 2 Convergence graph for 6-units with PD=700 MW

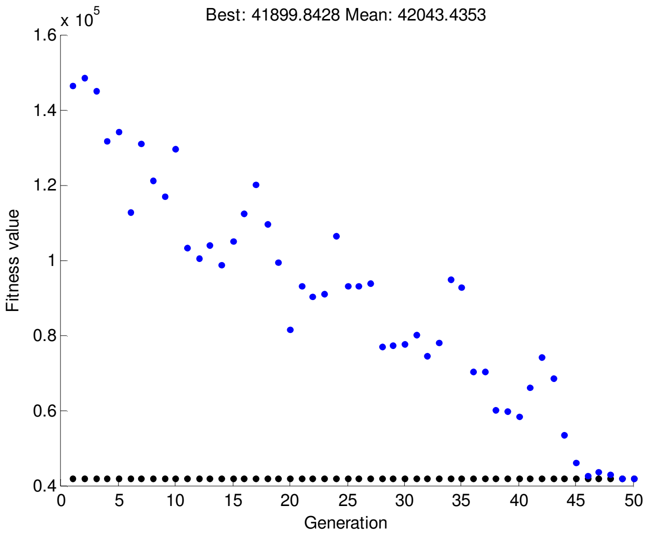

Fig. 3 Convergence graph for 6-units with PD=800 MW

It is clear from table II that the proposed PS method with elapsed time 9.689976 Seconds succeeded in finding a global optimal solution. While, GA succeeded to find optimal fuel cost in 7.336739 Seconds. Fig. 4 depicts the mesh size throughout the convergence process. It is apparent that the mesh size decreases until the algorithm terminates, in this case at a mesh size of $9.5367X10^{-7}$ in 9.689976 seconds.

Fig. 5 showed the convergence of the solution obtained by GA. The total of 80 iterations was spent during this process with elapsed time 7.336739 seconds. Tuned GA Options/Parameters for Case II (i.e. Generations: 80, Population size: 500, Crossover rate: 0.8 and Migration rate: 0.1).

TABLE II OPTIMAL SCHEDULES OF 40-UNITS WITH COMPARISON BETWEEN PS AND GA.

Unit	GA Algorithm		PS Method	
	Output MW	**Unit fuel Cost**	**Output MW**	**Unit fuel Cost**
1	108.76409	918.9593	113.99994	963.00738
2	114.00000	963.00789	114.00000	963.00789
3	117.63920	1430.34398	120.00000	1458.75712
4	190.00000	2281.39049	190.00000	2281.39049
5	97.00000	783.15995	97.00000	783.15995
6	140.00000	1583.69816	140.00000	1583.69816
7	300.00000	3045.77548	300.00000	3045.77548
8	300.00000	2955.91133	300.00000	2955.91133
9	300.00000	2975.59133	300.00000	2975.59133
10	136.56586	2598.31611	135.58815	2583.95048
11	94.78717	1904.34063	107.77335	2087.31320
12	94.38809	1913.60729	155.87152	2797.15703
13	127.91692	2581.78318	132.68083	2647.43165
14	311.14543	5272.98739	227.75405	4182.64085
15	282.76897	4910.60871	274.28204	4797.93304
16	203.20460	3894.29652	209.72029	3974.15634
17	500.00000	5466.41285	500.00000	5466.41285
18	500.00000	5458.25285	500.00000	5458.25285
19	550.00000	6034.26653	550.00000	6034.26653
20	550.00000	6034.24653	550.00000	6034.24653
21	550.00000	5387.85973	550.00000	5387.85973
22	550.00000	5387.85973	550.00000	5387.85973
23	550.00000	5370.57973	550.00000	5370.57973
24	550.00000	5370.57973	550.00000	5370.57973
25	550.00000	5598.19473	550.00000	5598.19473
26	550.00000	5598.19473	550.00000	5598.19473
27	14.03671	1170.08709	10.59584	1149.0008
28	11.97786	1159.55107	12.52935	1179.0573
29	11.30362	1159.55107	11.20464	1158.04425
30	97.00000	783.15995	97.00000	783.15995
31	190.00000	1523.74843	190.00000	1523.74843
32	190.00000	1523.74843	190.00000	1523.74843
33	190.00000	1523.74843	190.00000	1523.74843
34	200.00000	1917.97937	200.00000	1917.97937
35	200.00000	1860.68937	200.00000	1860.68937
36	200.00000	1860.68937	200.00000	1860.68937
37	107.50147	1136.87092	110.00000	1160.64994
38	110.00000	1160.64994	110.00000	1160.64994
39	110.00000	1160.64994	110.00000	1160.64994
40	550.00000	6034.26653	550.00000	6034.26653
P_{Demand}	**10,500.00 MW**		**10,500.00 MW**	
Best Total FC	**$119,732.25**		**$119,783.40**	
Time	**7.336739 Sec.**		**9.689976 Sec.**	

VII. CONCLUSIONS

This research work presents an application of PS and GA approaches for solving smooth and non-smooth ELD problems with valve-point effects. The two algorithms had been applied successfully to ELD with considering the effect of valve-point loading and relaxing all other equality and inequality constraints. The results obtained by both algorithms are very competitive and very close together. PS algorithm is very sensitive and dependant on single initial point which, may, possibly leads to be trapped in local minima. Nevertheless, PS succeeded in finding a global optimal solution and even competitive solutions. Moreover, GA always find solutions and these solutions may be changed from run to run as the algorithm normally uses initial population randomly. Other practical issues should be considered such as prohibited operating zone and multiple fuels as well. The later remains proposal to future research with integrating the two approaches for better performance.

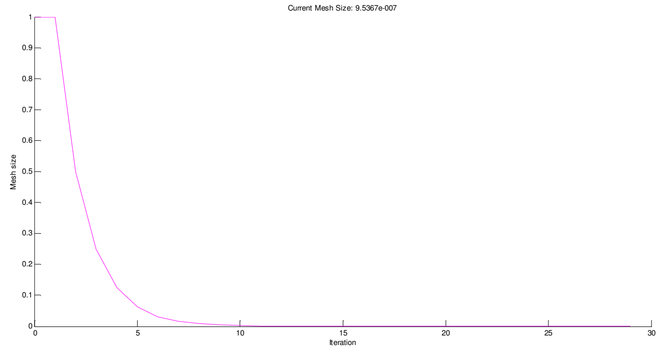

Fig. 4 Convergence of PS Mesh Size (Case II 40 Units)

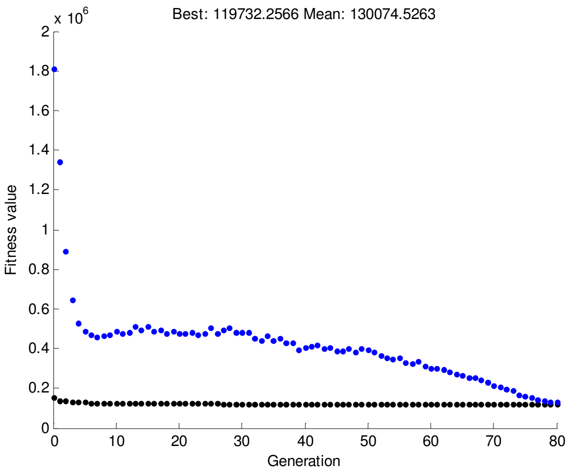

Fig. 5 Solution convergence by GA for 40 units

REFERENCES

[1] A.J Wood and B.F. Wollenberg, *Power generation operation and control*, John Wiley and Sons, New York, 2nd Edition, 1996.

[2] Hadi Sadaat., *Power system analysis*, International editions. WCB/McGraw-Hill. 1999.

[3] Jizhong Zhu, *Optimization of Power System Operation*, ISBN: 978-0-470-29888-6, Copyright © 2009 by IEEE.

[4] Saruhan, H., "Genetic Algorithms: An Optimization Technique", Technology, vol. 7, pp. 105-114, 2004.

[5] K. Y. Lee and M. A. El-Sharkawi (Editors), "Modern Heuristic Optimization Techniques with Applications to Power Systems", IEEE Power Engineering Society (02TP160), 2002.

[6] R. C. Bansal, "Optimization methods for electric power systems: An overview", Int. J. Emerging Elect. Power Syst., vol. 2, no. 1, pp. 1-23, 2005.

[7] Feng Gao, "Economic Dispatch Algorithms for Thermal Unit System Involving Combined Cycle Units", 15th PSCC, Liege, 22-26 August 2005.

[8] Amir Mohammadi, et al, "Online Solving Of Economic Dispatch Problem Using Neural Network Approach And Comparing It With Classical Method", IEEE—ICET 2006 2nd International Conference on Emerging Technologies Peshawar, Pakistan 13-14 November 2006.

[9] MIKAC, B., NKRET, R., "Application of a Genetic Algorithm to the Availability-Cost Optimisation of a Transmission Network Topology", Proceedings ICANNGA'97 Third International Conference on Artificial Neural Networks and Genetic Algorithms, Norwich, U.K., Springer Verlag Wien, New York, 1997, pp. 306–310.

[10] D. Attaviriyanupap, et al, "A hybrid EP and SQP for dynamic economic dispatch with nonsmooth incremental fuel cost function", IEEE Trans. Power Syst., vol. 17, no.2, pp. 411-416, 2002.

[11] N. Sinha, et al, "Evolutionary programming Techniques for economic load dispatch", IEEE Trans. Evol. Comput, vol. 7, pp. 83–94, Feb. 2003.

[12] S. Khamsawang and S. Jiriwibhakorn, "Solving the Economic Dispatch Problem using Novel Particle Swarm Optimization", International Journal of Electrical, Computer, and Systems Engineering, 2009.

[13] Z.L. Gaing, "Particle Swarm Optimization to Solving the Economic Dispatch Considering the Generator Constraints", IEEE Transactions on Power Systems, Vol. 18, No. 3, pp. 1187-1195 August 2003.

[14] Z. L. Gaing, "Constrained dynamic economic dispatch solution using particle swarm optimization", in Proc. IEEE Power Engineering Society General Meeting, pp. 153-158, 2004.

[15] J.B.Park , K.S.Lee , K.Y.Lee , "A Particle Swarm Optimization for Economic Dispatch with Non-smooth Cost Function", IEEE Trans. Power Systems, Vol 20 , Feb 2005.

[16] M. Sudhakaran, et al, "Application of Particle Swarm Optimization for Economic Load Dispatch Problems", The 14th International Conference on Intelligent System Applications to Power Systems, ISAP 2007, November 4 - 8, 2007, Kaohsiung, Taiwan, PP 668-674

[17] A. Bakirtzis, V. Petridis and S. Kazarlis, "Genetic algorithm solution to the economic dispatch problem", IEE Proc. Generation, Transmission and Distribution, vol. 141, no. 4, pp. 377-382, July 1994.

[18] P.H. Chen and H.C. Chang, "Large scale economic dispatch by genetic algorithm", IEEE Trans. Power Systems, vol. 10, no. 4, pp. 1919-1926, Nov.1995.

[19] W. Ongsakul, and N. Ruangpayoongsak, "Constrained dynamic economic dispatch by simulated annealing/genetic algorithms", in Proc. 22nd Power Ind. Computer Applications (PICA), Sydney, Australia, 2001, pp. 207-212.

[20] F. BENHAMIDA, "A Hopfield Solution to Economic Dispatch Problem with Transmission Losses", ACTA ELECTROTEHNICA, PP. 314-322, Volume 49, Number 3, 2008,

[21] R.S.Dhivy Apragash, et al, "An Efficient Particle Swarm Optimization for Economic Dispatch Problems with Non-smooth cost functions", WSEAS TRANSACTIONS on POWER SYSTEMS, ISSN: 1790-5060 Issue 4, Volume 3, April 2008, PP 257-266.

[22] Jong-Bae Park, et al.,"An Improved PSO for ED with Valve-point Effect", International Journal of Innovations in Energy Systems and Power , Vol. 1, no. 1 (Nov., 2006).

[23] Chen.P.H, and Chang.H.C, "Large-scale economic dispatch by genetic algorithm", IEEE trans. on Power Systems, Vol.10, N0.4, November, 1995, pp.1919-1926.

[24] http://www.mathworks.com.

[25] C. Thitithamrongchai, B. Euaarporn, "Self a adaptive Differential Evolution Based Optimal Power Flow for Units with Non-smooth Fuel Cost Functions", J. Electrical Systems 3-2 (2007), www.sciencedirect.com, pp.88-99.

[26] J.S. Al-Sumait et al. "Application of pattern search method to power system valve-point economic load dispatch", Electrical Power and Energy Systems 29 (2007), PP. 720–730

[27] Robert M. Lewis and Virginia T., "Pattern search algorithms for bound constrained minimization", SIAM Journal on Optimization, 9(4), PP. 1082-1099, 1999.

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